VISUALISATION IN MATHEMATICS PROBLEM SOLVING
META-ANALYSIS RESEARCH

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ABSTRACT

Visualisation among student must be enhanced in order to boost the knowledge insight in Mathematics. As for that reason, educational research must be conducted to overview on a vast transformation of visual usage on the Education System today to suit the 21st century learning environment. Solving problem practically would assimilate the student in conveying idea by collaborating people around them and immerse themselves to the real world more than just rote learning. The precious experience in this research discussion is to find out the way in visualise the student in solving problem especially in Mathematical fields. The transversal of Mathematical language to visualise drawing may become the potential method in enhancing the interpersonal skill and uphold the students’ comprehension. Expectant of this paper is to explore ideas and techniques used on visualisation in problem solving, thus contribution the approach to develop and improve the mathematics education especially in Malaysia.

Keywords: Mathematics problem solving, meta-analysis, visualisation.

1. Introduction

Mathematical problem sometimes gives a hectic to the student and they obviously feel terrible in solving it, thus lessen their interest in the lesson. We could find it quite familiar by evaluating the completeness of their homework. They struggle hard to understand the given mathematics problem and at the end, there were no solution is made. As stated in Ibrahim (1997), Lester & Kehle (2003) and Nunokawa (2005), a thinking process behind the entire mathematical program which involved learning of concepts and skills is used to solve the ambiguity problem on mathematics, thus successfully gain a new knowledge or experience in the situation. The problematic situation arise when the basic skills needed to solve the problem has not been mastered by the student (Bearch & Mazzocco, 2007; Tambychik & Meerah, 2010). However, this kind of problem can be solved by using and applying visualisation in mathematics classroom learning. The usage of word in problem solving plays an important role as solver components and develops the relationship with real life situation (Uesak et al., 2007).
The word problem solving in mathematics education plays prominent role in this presents. Ironically, solving problem using word seems to be very difficult if the students cannot do relation between the known and unknown especially when the student faced troublesome to understand the problem text given (Boonen et al., 2013). The comprehension of the student may refine by use of visualisation to simulate the student thinking varies rather than focusing on symbolism and formalism approach (Lavy, 2007). The powerful tools in learning mathematics is through visual which offer an alternative mass resources almost throughout the media as the representation of the simplified version of mathematical language especially in delivering the process of solving problem (Kosslyn, 1983; Lavy, 2007; Garderen 2006). This technics has been widely used in Singaporean dan Japanese School curricular focusing on the elementary school as the basis of exposure to the mind of creativity and criticist (Beckmann, 2004; Murata, 2008). As the result, communication of mathematical ideas using visual such as tape diagram and simple picture aiding the student in connecting ideas across the problem given (Cheong, 2002; Ho & Lowrie, 2014). Moreover, the improvement in tackling technics of the problem in mathematics improved the skill of thinking among students. The aim of this study is to explore ideas and techniques used on visualization in problem solving, thus contribution the approach to develop and improve the mathematics education especially in Malaysia.

2. Role of Visual Representation in Problem Solving

The level of learning process is divided into three which include enactive, iconic and symbolic. Enactive is the crucial level of visualisation which performs the connection between the practices and formal level of understanding or in other word the mediator of the communication (Bruner 1961; Deliyyanni et al. 2009). Diagram or picture that the student use or construct to enhance their understanding will automatically generate a big picture in their mind to dig up the solution of the problem (Deliyyanni et al., 2009; Rosken & Rolka, 2006). It does not only helping them to establish a relationship of mathematical images but also an effective way in solving any problem syntactically, semantically and pragmatically perspective (Brown & Wheatley, 1997). The well-form use of pictorial signs will make a good perspective of syntactic, meaningful used of pictorial signs will show semantic perspective while a pictorial signs that being used to think, communicate and learn will give a pragmatic perspective (Schnotz, 2002). According to Carney & Levin (2002), there are five functions of pictures serve with text processing referring to pragmatic perspective.

The five function pragmatic perspective are Transformational, Organizational, Decorative, Interpretational and Representational shown in as Table 1 below.

Table 1: Functions of Pragmatic Perspective

<table>
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<tr>
<th>Pragmatic Perspective</th>
<th>Meaning</th>
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<tr>
<td>Transformational pictures</td>
<td>A component of mnemonic system to enhance the student recall memory of information especially text.</td>
</tr>
<tr>
<td>Organizational pictures</td>
<td>Structural framework that is handy for the text content</td>
</tr>
<tr>
<td>Decorative pictures</td>
<td>More for designation of page than relating to text content</td>
</tr>
<tr>
<td>Representational pictures</td>
<td>Illustration of a part or the whole text content</td>
</tr>
<tr>
<td>Interpretational pictures</td>
<td>Comprehends the understanding of difficult question text</td>
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</table>
Based on Table 1 above, Schnotz (2002) found that students could remember information better when text is illustrating together with a picture. Advanced mathematical according to the researchers also benefit from the visual representation in solving problem such as geometry, algebra, probability and others. Compare to the novice mathematician, the expert frequently use images to visualise approach in building understanding on the situation of problem and guide them to plan for the solution (Stylianou & Silver, 2004).

3. Methods and Procedures

Sample in this study were previous papers which cover on visualisation in problem solving. There were 52 papers gathered at first, however only 26 papers are selected since the other 26 studies had to be eliminated as they were not fulfil the requirement needed to conduct the meta-analysis. For the papers to be selected, they must feature following characteristics, which are: (1) Visual problem solving in mathematics education, (2) Diagram in visual problem solving, (3) Model/tape diagram in visual problem solving, (4) Visualise and non-visualise method in the basis of mathematics, (5) Visual representation in mathematical world problem solving and (6) Difficulties using visual representations. Those papers were searched in commonly used electronic databases including SpringerLink, ScienceDirect, Taylor & Francis, SAGE, ERIC, Wiley Online Library and Google Scholar via the following keywords like visualisation in problem solving, visual representation in mathematics education, and model/tape diagram in visualisation problem solving.

4. Findings and Discussion

A total of 26 studies on visualisation have been chosen to be presented in meta-analysis. Through the meta-analysis conducted, we managed to identify several aspects that can help us to implement visualisation in problem solving. The aspect that we chose from the 26 studies are learning disabilities using diagram, nonvisual vs visual, performance students, and external visual representation.

4.1 Learning Disabilities and Using Diagram

Many studies have proven and demonstrated the beneficial effect of visualisation in problem solving by representative of diagram (Ainsworth & Th Loizou, 2003; Cheng, 2004; Pedone, Hummel, & Holyoak, 2001). However, students with Learning Disabilities (LD) also benefit from this method as such some researcher implement visual imagery as a supportive element to solve mathematical word oriented problems. As mention by Van Garderen (2006), some LD students would struggle to use visualisation as a strategy and at the end they make a better impact in their mathematics performance. Diagrams somehow is the powerful tools when facing solving word orientation problems (Uesaka, Manalo & Ichikawa, 2007) especially when use it with a systematic way. Furthermore, it is not only substantially improve achievement the LD students but also (Butler et al., 2003) it is the most effective strategies that have been proposed to improve efficiency in mathematical problem solving (Uesaka, Manalo & Ichikawa, 2007). However, before LD students become proficiency in using diagram to solve word problems, there have to face challenges of five strands on the “diagram proficiency” which are shown in Table 3.
Five strands shown in Table 3 is not a major problem when involving students with higher achiever because making diagram according to word problems given with specific strategy is easy-peasy for them. Structural representations with simple surface are consider as a diagram which mean that students can develop diagrams based on their understanding on word problem given (Vekiri, 2002). However, the problem in making the right diagram is happen to LD students. They try to make perfect diagram as mention in Table 3, but the problem occur when they cannot digest and understand all the information given in word problems perfectly (Rosken & Rolka, 2006).

Table 3: Strands of diagram proficiency (Note. Adapted from NRC, 2001 p. 117)

<table>
<thead>
<tr>
<th>Strand</th>
<th>Description</th>
<th>Abilities</th>
</tr>
</thead>
<tbody>
<tr>
<td>Conceptual Understanding</td>
<td>Comprehension of what relationships a diagram can represent and how a diagram can be used when solving a problem.</td>
<td>Can describe what a diagram is and the ways a diagram can be used for solving a problem.</td>
</tr>
<tr>
<td>Procedural Fluency</td>
<td>Skill in accurately and efficiently generating a diagram that represents the problem situation.</td>
<td>Can generate a diagram that represents the problem situation.</td>
</tr>
<tr>
<td>Strategic Competence</td>
<td>Ability to represent and use a diagram to solve a problem.</td>
<td>Can use a relevant diagram as a tool to represent, solve and monitor the problem solving process.</td>
</tr>
<tr>
<td>Adaptive Reasoning</td>
<td>Ability to justify and communicate an explanation of one’s use of a diagram to solve a problem.</td>
<td>Can explain and justify how the diagram that was used was appropriate to help solve the problem.</td>
</tr>
<tr>
<td>Productive Disposition</td>
<td>See a diagram as beneficial, worthwhile and sensible for solving a problem coupled with a belief and confidence in one’s own ability to use it.</td>
<td>Recognizes when a diagram can be beneficial for solving a problem and will self-initiate use, with confidence, as needed.</td>
</tr>
</tbody>
</table>

The achievement whether getting success or failure in using diagrams correctly due to students’ inability and least experience of using diagrams in problem solving (Pantziara, Gagatsis & Elia, 2009; Tambychik & Meerah, 2010). They have to learn and understand the text of problem given before successfully sketching up the diagrams (Diezmann & English, 2001, Pantziara et al., 2009). According to Garderen (2007), the main reason for least performance of mathematics is the poor conceptualisation understanding. The limited diagram proficiency leads to poor problem solving (Diezmann, 2007).

Representation of the problem structure to diagram depends to its appropriateness because it will be used as in the process to get the solution of the question (Booth & Thomas, 1999; Pantziara et al., 2009). There are three differences kind of difficulties on visualisation faced by student with LD that is (1) frequency of using any visual representation strategies in solving problem is very low, (2) the quality of diagram and the interrelationship with the problem statement is very poor and (3) they use very limited visual strategies in problem solving such as organise, plan, monitor, compute and justify (Garderen & Montague, 2003). All those weakness showed by LD students is not a major
problem as these can be fixed with some teaching and learning on how to construct and proficiency the diagram.

For example, instruction may be needed to help students in develop a comprehension on the usage of difference kind of diagram in order to tackle the question needs, thus solve any problem in mathematics flawlessly especially in geometry and analytic (Pape & Tchoshanov, 2001). Classroom with visual-spatial students need to be explicitly instructed when come to problem solving situation (Ho & Lowrie, 2014; Pape & Tchoshanov, 2001). Teaching a student to draw a diagram without educating them thinking or getting them to imagine about the problem will make visual tool look useless. (Diezmann 2000; Garderen et al. 2014).

4.2 Visual vs Non-Visual

From 26 studies that we choose, all of the findings show that the implementation of visualisation helps in problem solving process. Some of the students much more preferred to use visual method when solve difficult problems (Uesaka, Manalo, & Ichikawa, 2007; Ho & Lowrie, 2014) and for less difficult problems, they tend to use nonvisual method (Lowrie, 2010; Ho & Lowrie, 2014) while those student with high spatial ability tended to use their ability to manipulate 2D and 3D in their mind (spatial skills) while solving a problem (Booth & Thomas, 1999). By using visualisation, we can see the students’ understanding of the problems before they solve the problems (Nunokawa, 2005). However, non-visual students look visualisation method as not giving any benefits for them because it requires them to well known how to use a tool like sketching diagram or picture in solving a problem. Moreover, they need to collect and spindle up all the information from other source to understand the whole problem globally (Elia & Philippou, 2004). As they feel that it is so difficult to figure out, they reluctant to use them in the process of solving problem (Uesaka et al., 2007). They prefer to use algebraic method to process the solution even though they faced complicated problem (Stylianou & Silver, 2004).

For the higher achiever, the visualisation method become a choice for them when solve a problems as they already know how to construct well diagram as mention in Table 3 (Garderen et al. 2012). Integration of text and visual elements to represent the solution at the tips of their finger (Boonen et al., 2013). As for that, the visual representation give an enormous contribution solution spontaneously and functionally to the world of mathematics whether non-routine and routine questions (Deliysianni et al. 2009).

4.3 Academic Performance

There are three studies (Beckmann, 2004; Murata, 2008; Uesaka et al., 2007) that analyse the students’ performance when using visualisation method. Beckmann (2004) compares the ranking of students 8th graders between U.S. and Singapore on TIMSS (Trends in International Mathematics and Science Study) where the result shows that higher score goes to Singapore 8th graders. The textbook visualisation presentations of Singapore’s elementary school are very interesting with accessible method of problem solving integration (Ho & Lowrie, 2014). By using simple diagram and pictures, it can help the students to understand the problems before come out with the solution. Cheng (2004) points out that solution assisted with picture or diagram six times more powerful than sentential type of solution and it built the strong mastery in problem solving. While Murata (2008) which investigating the Japanese teaching and learning strategies said that tape diagram or bar model
implemented inside their textbook relate the quantities in the problem given. Singapore mathematics textbooks also implement this method in their curriculum and the students show a great performance in their study. Beside tape diagrams, there are other visual representation uses in Japanese textbooks such as ten frames, number lines, base-ten blocks and pictures. Both countries had shown a great performance when implementing visual representation in their curriculum. Uesaka et al. (2007) comparing students’ performance between New Zealand and Japanese in PISA (Programme for International Student Assessment) which assesses the skill and knowledge student at the age of 15 and found that both countries get highly ranking in 2000 and 2003 although New Zealand had higher score rank than Japan. The establishment of the student mathematical knowledge comes from the encouragement of using diagram as a tool in solving problem. Japanese curricular more emphasizing more on understanding diagrams and mathematical concept rather than assuming the diagram as a tools as such the New Zealand national curricular does to their student in encouraging the usage of the tool spontaneously in problem solving.

4.4 External Visual Representation

Visual representation can be classified into two categories which are external and internal. Cognitive schemata or mathematical ideas pictures develop by student through their experience are called internal visual representations whereas manifestations of mathematical concepts which assist the student in understanding concepts including symbolic system of mathematics are called external representation. Table 4 below show the external visual representation type found in previous studies (Goldin & Shteingold, 2001; Ho & Lowrie, 2014; Pape & Tchoshanov, 2001).  

<table>
<thead>
<tr>
<th>Table 4: Types of external visual representation</th>
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<tbody>
<tr>
<td>Non-diagrammatic</td>
</tr>
<tr>
<td>Reorganization</td>
</tr>
<tr>
<td>Outcome Listings</td>
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Based on Table 4, external visual representation use to solve probability problems can be summarizing into 3 categories. The first category is non-diagrammatic which include reorganization and outcome listings. The second category is schematic diagrams which include Novel Schematic Trees Diagrams, Venn diagrams, and Contingency Tables. Lastly, pictures belong to the third category, iconic. Seven variety of external visual representation are differing from each type. The first group (non-diagrammatic) used to compare information in the form of table in order organise the item lists. Next, the second group diagrams are used to represent of problem by graphing schematic structure or syntax to get the big picture of the question before calculating the result. Lastly, the picture can be use as the iconic problem representation in order to give understanding before the student could interpret or predict the solution. (Zahner & Corter, 2010). In problem solving, finding a solution can be done by implementing representation the problem by external
visualisation. Deliyianni et al. (2009) find out that visualisation in solving problems were influence by the contract rules on pupils. The influences become stronger as the students get older. By referring to that finding, visualisation in solving mathematical problems is depends on the students’ age.

Student use creative visualisation in solving process in pre-school education, but no longer use the visualisation after getting in primary school as at the time, teacher rarely use visualisation approach in teaching solving mathematical problems. Presmeg & Balderas-Cañas (2001) and Macnab et al. (2012) studied found that teacher playing an active role in pedagogy at all levels in the selection of visualisation object as it influenced students in choosing visualisation method or not in their skill in solving problem.

5. Conclusion

As the conclusion, visualisation which combines skill interpretation of drawing and problem solving has a potential to increase credibility and potential among the student. The approach requires students to think creativity and critically in order to answer for every questions posted by the teacher. Previous research in Malaysia has fewer ventures to integrate the visualisation in problem solving in depth as explained earlier in this paper. Therefore, the effectiveness of the combination in increasing comprehension among the students has yet to be examined.

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