STATE ESTIMATION FOR FUZZY SYSTEM DIAGNOSIS:
APPLICATION TO A BIOLOGICAL REACTOR

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Abstract
This paper deals with a robust method for state reconstruction of nonlinear systems subject to unknown inputs. The multiple model methodologies can contribute significantly to the achievement of this objective since they can provide accurate approximation properties. We focus on the design of observers for dynamic Takagi-Sugeno fuzzy systems that can be used in a model-based fault detection and diagnosis scheme. A nonlinear system of this class is composed of multiple affine local linear models that are smoothly interpolated by weighting functions resulting from a fuzzy partitioning of the state space of a given nonlinear system subject to observation. The design methodology of the so-called fuzzy observers for fault detection and diagnosis is based on extending Luenberger observers schemes to the case of interpolated multiple models. Stability conditions are performed by using Linear Matrix Inequalities (LMIs). An application to a biological reactor is given.

Keywords: Takagi-Sugeno fuzzy models, Sliding Mode Observers, State Estimation, Fault diagnosis, Biological Reactor

1. Introduction

Nowadays, there is a growing demand for higher performance as well as for more safety and reliability of dynamic systems. In order to satisfy this demand, more advanced systems equipped with a large number of sensors, actuators and other components are being used. Such a complexity results in an increased probability of failure. Meanwhile, stimulated by the strong desire for improving the reliability and performance of systems, many research efforts in the field of process supervision, fault detection and diagnosis have been made (Isermann, 1997; Gertler, 1998; Patton, 2000). One of the most promising techniques for solving this challenging task is the analytical approach, in which some useful symptoms or residual signals, which reflect inconsistencies between nominal and faulty system states, are obtained by using the input and output signals and applying dynamic process models.

Modern approaches are based on a process model and exploit the mathematical relations between different process signals (Chen, 1999; Patton, 2000; Simani, 2003). They enable a fine diagnosis but require deeper insight and understanding of the process and need much effort to develop, particularly for nonlinear and complex processes. Obviously, they require accurate mathematical models of the plant. However, the task of establishing a mathematical description for complex nonlinear processes is often difficult and time consuming.

In recent years, the monitoring and the diagnosis of nonlinear processes has received an increasing interests from the scientific and engineering practitioners. In general, the nonlinear systems are firstly linearized at an operating point, and then robust techniques are applied to generate residuals, which are robust against limited parameter variations. The strategy only works well when the linearization does not cause a large mismatch between linear and nonlinear models; when the system operates near the specified operating point. Therefore such techniques have limited robustness when considering gross plant changes and nonlinearity.
In the lack of first-principle models, empirical models like neural networks can be used for the purposes of process supervision. The main problems with these approaches are the difficulty in analyzing, in a rigorous mathematical way, their robustness/sensitivity and the scalability; i.e a network trained for a specific plant may be inappropriate for other plant. To overcome the problem of precision and accuracy in FDD, various approaches based on fuzzy logic have been also suggested (Ayoubi, 1997; Ballé, 1999, 2000; Isermann, 1998).

However, the fuzzy logic approach is not only required on its own, but as a framework for combining different paradigms. More specifically, quantitative model-based and soft-computing are combined to exploit the benefit of each (Frank, 1997; Patton, 1997). Another powerful approach for residual generation is based on observers. The common way is to obtain a set of residuals by comparing the actual measurements with their estimates obtained with the help of observers. Unfortunately, the design of nonlinear observers is not a straightforward task, even if the nonlinear process is completely known.

In this paper, we propose a methodology for the diagnosis of dynamic nonlinear processes by combining fuzzy logic with model-based methods to formulate the so-called fuzzy observers. Typically, the design of a fuzzy observer requires a precise mathematical description of the plant under interest in the form of a fuzzy dynamic model, which includes both local linear models and fuzzy membership functions. The local linear models are state space affine models that can be derived directly from first principle or from empirical models (Johansen, 2000; Nelles, 2001).

This paper is organized as follows: section 2 outlines the multiple modeling methodology of dynamic nonlinear processes. In section 3, the design of fuzzy observers is treated and in particular the fuzzy sliding mode observers. The description of process components and the simplified mathematical model of the biological reactor are given in section 4. Simulation studies concerning the biological nutrient removal process are presented in section 5. Finally, some concluding remarks as well as some possible improvements are given in section 6.

2. Multi-linear models

The major motivation for the multiple modeling methodologies is that locally there are less relevant phenomena, and interactions are simpler. Typically, this is done by dividing the full range of all possible operating conditions into several regimes where in each regime the system is represented by local linear models. Obviously, it is assumed that the whole operating range of the system is completely covered by these regimes. At some operating regions, there may be an overlap between regimes where several models may be valid. To address this issue, a variety of approaches are available. One useful approach is to associate a validity function to each local linear model. Then, the overall nonlinear process is approximated by combining the local models into a single global model. For the purposes of this paper, we assume that the local linear models are related to local linearization, via Taylor series expansion, of the original nonlinear system at off-equilibrium points (Johansen, 2000)

2.1. model structure

Consider a nonlinear system of the form

\[
\begin{align*}
\dot{x} &= h(x,u) \\
y &= g(x)
\end{align*}
\] (1)
where the state vector $x \in \mathbb{R}^n$, the control vector $u \in \mathbb{R}^m$, the output vector $y \in \mathbb{R}^p$, and the functions $h$ and $g$ are nonlinear functions. In the following, we assume that (1) can be represented or approximated sufficiently well by a Takagi-Sugeno (TS) fuzzy system. The TS system consists of a fuzzy rule base, where the $i$th rule can have the form:

$$\text{RULE}_i: \text{IF} z_i \text{ is } A_i \text{ and ... and } z_r \text{ is } A'_i \text{ THEN} \begin{cases} \dot{x} = A_i x + B_i u + a_i \\ y = C_i x + c_i \end{cases} (2)$$

where the vector of premise variables $z \in \mathbb{R}^r$ is a subset of $x, u, \theta$ and $y$ and $A'_i$ is a fuzzy subset of membership function $\mu_{A'_i}: \mathbb{R} \rightarrow [0,1]$. The function $\mu_{A'_i}(z_j)$ denotes the $j$th membership function in the $i$th rule which applies to the $j$th premise variable. The Cartesian product $\mu_{A'_i}(z_i) \times \ldots \mu_{A'_i}(z_r)$ defines a fuzzy region in $\mathbb{R}^r$. The system matrices $A_i$ and $B_i$ may be obtained by a linearizing transformation or through Taylor expansion in some point $(x_i, u_i)$ corresponding to $z_i$ in fuzzy region described by each rule $i$

$$A_i = \left. \frac{\partial h}{\partial x} \right|_{(x_i, u_i)}, B_i = \left. \frac{\partial h}{\partial u} \right|_{(x_i, u_i)}, C_i = \left. \frac{\partial g}{\partial x} \right|_{(x_i)}$$ (3)

Note that $(x_i, u_i)$ does not have to be an equilibrium point which means that constant or affine terms in each subsystem $i$ may be obtained as

$$a_i = h(x_i, u_i) - A_i x_i - B_i u_i$$
$$c_i = g(x_i) - C_i x_i$$

The global TS system described by (2) is then written as

$$\begin{cases} \dot{x} = \sum_{i=1}^{M} w_i(z)(A_i x + B_i u + a_i) + f(t, x, u) \\ y = \sum_{i=1}^{M} w_i(z)(C_i x + c_i) \end{cases} (4)$$

where $M$ is the number of rules and

$$w_i(z) = \frac{\mu_i(z)}{\sum_{k=1}^{M} \mu_i(z)} \mu_i(z) = \prod_{j=1}^{r} \mu_j(z_j)$$

with $\sum_{i=1}^{M} w_i(z) = 1$ and the term $f(t, x, u)$ encapsulates any uncertainty or nonlinearities and it is assumed to be unknown but bounded. Furthermore, we consider the case $y = Cx$ which covers a large class of real technical systems.
3. Fuzzy sliding mode observer

Sliding mode observers are known for their robustness and insensitivity with respect to many kinds of uncertainty. These observers are more robust than Luenberger observers, as the discontinuous term enables the observers to reject disturbances, and also a class of mismatch between the system and the observer. The discontinuous term is designed so that the state estimation error vector remains on a surface in the error space. The induced motion is referred to as the sliding mode (Utkin, 1992). In most cases, the sliding surface is the difference between the observer and system output which is therefore forced to be zero. Several authors have proposed sliding mode observers design methods (Edwards, 2000; Tan, 2001; Walcott & Zak, 1987). The method of Walcott and Zak (Walcott & Zak, 1987) requires a symbolic manipulation package to solve the design problem which is formulated. Edwards and Spurgeon (Edwards, 2000) proposed a canonical form for sliding-mode observer design and they give a numerically tractable algorithm to compute the gain matrices and the state transformation matrix to obtain the canonical form. The observer developed in this section is based on ideas found in (Akhenak, 2003). The main idea behind it is to extend the traditional sliding mode observers to dynamical systems described by a multiple model. In this context, the design of fuzzy Luenberger observers are proposed in (Tanaka, 1997), and lately applied for the detection and isolation of faults in nonlinear dynamic systems (Patton, 1998).

Assume that the fuzzy approximation of a nonlinear system reads

$$\dot{x} = \sum_{i=1}^{M} w_i(z)(A_i x + B_i u + a_i) + f(t, x, u)$$

$$y = Cx$$

and the following assumptions are satisfied:

A1. $$f(t, x, u) = R\bar{u}(t)$$

A2. $$R = \sum_{i=1}^{M} w_i(z)R_i$$

A3. $$\bar{u}(t) \in \mathbb{R}^q$$, $$R_i \in \mathbb{R}^{n \times q}$$, and $$C \in \mathbb{R}^{p \times n}$$ with $$p \geq q$$.

The function $$\bar{u}(t)$$ represents the matched uncertainty due to the existence of unknown inputs. For the sake of simplicity, $$\bar{u}(t)$$ is denoted by $$\bar{u}$$.

$$\begin{align*}
\begin{cases}
\dot{x} = \sum_{i=1}^{M} w_i(z)(A_i x + B_i u + R_i\bar{u} + a_i) \\
y = Cx
\end{cases}
\end{align*}$$

such that:

$$\begin{cases}
\sum_{i=1}^{M} w_i(\xi) = 1 \\
0 \leq w_i(z) \leq 1 \quad \forall i = \{1, \ldots, M\}
\end{cases}$$

where $$x(t) \in \mathbb{R}^n$$ is the state vector, $$u(t) \in \mathbb{R}^m$$ is the input vector, $$\bar{u}(t) \in \mathbb{R}^q$$, $$q < n$$, the vector of unknown inputs and $$y(t) \in \mathbb{R}^p$$ the vector of measurable output. For the $$i$$th local model $$A_i \in \mathbb{R}^{n \times n}$$ is the state matrix, $$B_i \in \mathbb{R}^{n \times m}$$ is the matrix of inputs, $$R_i \in \mathbb{R}^{n \times q}$$ is the matrix of influence of the
unknown inputs, \( a_t \in \mathbb{R}^{n_x} \) is the offset vector which depends on the operating point and \( C \in \mathbb{R}^{n_y\times n} \) is the matrix of output. Finally, \( z \) represents the scheduling vector which is formed by a subset of the input and/or the measurable state variables to define the validity regions of the local models.

The problem considered here consists in the reconstruction of the state variables by using the information provided by the input and output signals and in the cases of sliding mode the reconstruction of faults is also treated. The proposed observer for the multiple model (6) is a linear combination of local observers, each of them having the structure proposed by Walcott and Zak. In this context, we consider that the inputs \( \vec{u}(t) \) are bounded, such as \( ||\vec{u}(t)|| \leq \eta \), where \( \eta \) is scalar and \( ||\cdot|| \) represents the Euclidean norm. It is also assumed that there exists matrices \( G_i \in \mathbb{R}^{n_y \times p} \), such that \( A_{ii} = A_i - G_i C \) have stable eigenvalues and that there exists Lyapunov pairs \((P, Q_i)\) of matrices and other matrices \( F_i \in \mathbb{R}^{p \times p} \) respecting the following structural constraints:

\[
\begin{align*}
A_{ii}^T P + PA_{ii} &= -Q_i \\
F_i C &= R_i^T P, \quad \forall i \in 1, \ldots, M
\end{align*}
\] (7)

The proposed observer has the form:

\[
\begin{align*}
\dot{x} &= \sum_{i=1}^{M} w_i(z) \left( A_i \dot{x} + B_i \mu + a_i + G_i e_y + R_i v_i \right) \\
y &= C \dot{x}
\end{align*}
\] (8)

where \( e_y \) is the output error defined as follows:

\[
e_y = y - \hat{y} = C (x - \hat{x}) = Ce
\] (9)

with \( e(t) \) represent the state estimation error, such as:

\[
e(t) = x(t) - \hat{x}(t)
\] (10)

The matrices \( G_i \) and the control variables \( v_i \), with \( v_i(t) \in \mathbb{R}^y \) must be determined in order to guarantee the asymptotic convergence of \( \dot{x}(t) \) towards \( x(t) \). The terms \( v_i(t) \) compensate errors due to the unknown inputs. The dynamic of state estimation error is given as follows:

\[
\dot{e} = \sum_{i=1}^{M} w_i(z) \left( (A_i - G_i C) e + R_i \vec{u} - R_i v_i \right)
\] (11)

**Theorem 1:** The state estimation error between the multiple model (6) and the robust state multiple observer (8) converges towards zero, if \( v_i(t) \) are given by the following equation:

\[
\begin{align*}
\text{if } e_y(t) \neq 0, \text{ then } v_i(t) &= \eta \frac{F_i e_y}{||F_i e_y||} \\
\text{if } e_y(t) = 0, \text{ then } v_i(t) &= 0
\end{align*}
\] (12)
and if there exists a symmetric definite positive matrix \( P \) which satisfies the following inequalities:

\[
(A_i - G_i C)^T P + P(A_i - G_i C) < 0, \quad i = \{1, \ldots, M\}.
\]  

\[(13)\]

**Proof:** In order to show the asymptotic convergence of this multiple observer, let us consider the following Lyapunov function:

\[
V(e(t)) = e^T(t)Pe(t)
\]

Its derivative in respect to time, evaluated along the trajectory of the system by using equations (9) and (11), may be expressed as:

\[
\dot{V} = \sum_{i=1}^{M} w_i(z) \left( e^T \left( \bar{A}^T_i P + P \bar{A}_i \right) e + 2e^T PR_i \bar{u} - 2e^T PR_i v_i \right)
\]

where \( \bar{A}_i = A_i - G_i C \).

Using the second part of constraint (7), the derivative of the Lyapunov function becomes:

\[
\dot{V} = \sum_{i=1}^{M} w_i(z) \left( e^T \left( \bar{A}^T_i P + P \bar{A}_i \right) e + 2e^T F_i^T \bar{u} - 2e^T F_i^T v_i \right)
\]

\[
= \sum_{i=1}^{M} w_i(z) \left( e^T \left( \bar{A}^T_i P + P \bar{A}_i \right) e + 2e^T F_i^T \bar{u} - 2e^T F_i^T v_i \right)
\]

\[
\leq \sum_{i=1}^{M} w_i(z) \left( e^T \left( \bar{A}^T_i P + P \bar{A}_i \right) e + 2\eta \| F_i^T \| \bar{e}^T \| F_i^T \| v_i \right)
\]

Using the relation (12), the derivative of the Lyapunov function becomes as follows:

\[
\dot{V} \leq \sum_{i=1}^{M} w_i(z) \left( e^T \left( \bar{A}^T_i P + P \bar{A}_i \right) e + 2\eta \| F_i^T \| \bar{e}^T \| F_i^T \| v_i \right)
\]

\[
\dot{V} \leq \sum_{i=1}^{M} w_i(z) \left( e^T \bar{A}^T_i P + P \bar{A}_i e \right)
\]

\[(16)\]

Then, the state estimation error of the robust multiple observer (8) converges towards zero if the relation (13) holds.

4. **Dynamical model of the biological reactor process**

An activated sludge bioprocess belongs to a class of an aerobic wastewater treatment process. It is mainly constituted by two tanks, Fig. 1: The Aerated Tank is a biological reactor containing bacteria and other micro-organisms population grown in order to remove the organic matter contained in the incoming wastewater by biodegradation, and the clarifier (a gravity settlement tank) where the sludge and the clean effluent are separated. A part of the removed sludge is recycled back to the aeration tank and the other part removed (Crisan, 2011a; Vlad, 2011).
A reduced nonlinear model named the Activated Sludge Model No. 1 (ASM1) of the International Water Association (IWA) is used in this article to describe the dynamical behavior of the bioprocess (Henze, 2000; Nejjari, 1999).

The following hypotheses are considered in model formulation:

**H1**: The settler is considered working perfectly, i.e. there is no biomass or dissolved oxygen in the recycled sludge flow of the bioreactor;

**H2**: The system runs in steady-state regime \( F_{in} = F_{out} = F, \ D = F/V \);

**H3**: The recycled sludge is proportional to the process flow \( F \): \( F_r = r \cdot F \), where \( r \) is the recycled sludge rate;

**H4**: The flow of the sludge removed from the bioreactor (sludge that is in excess) is considered proportional to the process \( F \): \( F_{\beta} = \beta \cdot F \), where \( \beta \) is the removed sludge rate;

**H5**: The exit flow rate from the aeration tank equals the sum between the exit flow rate from the clarifier tank (settler) and the recycled sludge flow.

Then the dynamics of the plant are described by the following mass-balance equations:

\[
\frac{dX}{dt} = \mu(t) X(t) - D(t)(1+r)X(t) + rD(t)X_r(t) \quad (17a)
\]

\[
\frac{dS}{dt} = -\frac{\mu(t)}{Y} X(t) - D(t)(1+r)S(t) + D(t)S_{in} \quad (17b)
\]

\[
\frac{dDO}{dt} = -K_0 \frac{\mu(t)}{Y} X(t) - D(t)(1+r)DO(t) + \alpha W(\max(DO_{\text{max}}) - DO(t)) + D(t)DO_{in} \quad (17c)
\]

\[
\frac{dX_r}{dt} = D(t)(1+r)X(t) - D(t)(\beta + r)X_r(t) \quad (17d)
\]

where \( X, \ S, \ DO \) and \( X_r \) denote the biomass, the substrate, the dissolved oxygen and the recycled biomass, respectively. A possible structure of the nonlinear specific growth rate \( \mu(t) \) is a Monod-type model:

\[
\mu(t) = \mu_{\text{max}} \frac{S(t)}{k_s + S(t)} \frac{DO(t)}{K_{DO} + DO(t)} \quad (18)
\]
The other variables present at the model are:

- dilution rate: \( D(t) \)
- aeration rate: \( W \)
- substrate and dissolved oxygen concentrations in the influent: \( S_{in} \) and \( DO_{in} \)

4. Simulation results

The following parameters were used in model described earlier (Table 1):

<table>
<thead>
<tr>
<th>Symbol</th>
<th>Meaning</th>
<th>Value</th>
</tr>
</thead>
<tbody>
<tr>
<td>( Y )</td>
<td>Biomass yield factor</td>
<td>0.65</td>
</tr>
<tr>
<td>( \beta )</td>
<td>Ratio of waste flow to the Influent</td>
<td>0.2</td>
</tr>
<tr>
<td>( \alpha )</td>
<td>Oxygen transfer rate</td>
<td>0.018</td>
</tr>
<tr>
<td>( K_{DO} )</td>
<td>Saturation constants</td>
<td>2 mg/l</td>
</tr>
<tr>
<td>( K_0 )</td>
<td>Model constants</td>
<td>0.5</td>
</tr>
<tr>
<td>( \mu_{max} )</td>
<td>Maximum specific growth rate</td>
<td>0.15mg/l</td>
</tr>
<tr>
<td>( K_s )</td>
<td>Saturation constants</td>
<td>100mg/l</td>
</tr>
<tr>
<td>( DO_{max} )</td>
<td>Maximum Dissolved oxygen</td>
<td>10mg/l</td>
</tr>
<tr>
<td>( r )</td>
<td>Ratio of recycled flow to the influent</td>
<td>0.6</td>
</tr>
</tbody>
</table>

The TS fuzzy model is formed by four IF-THEN logical rules that have a fuzzy antecedent part and a functional consequent part. Concerning the dynamics of the bioprocess and its nonlinear model structure, two fuzzy variables are considered in the antecedent part of the TS fuzzy model: the dilution rate and the aeration rate. The membership functions of the scheduling variables are depicted in figure 2. The local dynamic models are deduced from the nonlinear model (1) through dynamic linearization about operating points.
Figure 2: Membership function of fuzzy scheduling variables

State estimation

To see the effectiveness of the proposed SMUIO, simulation results are presented in figures 3 and 4. Figure 3 shows the unknown inputs presented to the system while Figure 4 shows the state estimation. Based on Figures 1 and 2, it can be seen that the observer performs as expected despite the presence of the unknown inputs and the real and estimated states are found to be close.

Figure 3: The unknown inputs
Sensor fault detection

In order to identify the sensor fault, we design a global observer based on the two outputs $y_1 = S$ and $y_2 = DO$. The sensor fault detection and isolation is based on the analysis of the residuals generated by the observer, depend on two inputs applied to the system (17). The additive signal that represents sensor failure has been added to sensor output $y_1$ between 60h and 80h. The simulation results of the fault detection and isolation based on the global observer are illustrated on the figures 5 and 6. The residuals ($r_1$) and ($r_2$) allow to detect and locate the fault sensor on the substrate output $S$. 

Figure 4: The four states: Dotted line: estimated state; solid line: real state
6. Conclusion

A suitable sliding mode observer for dynamic nonlinear processes subject to unknown inputs is developed in this paper. The nonlinear system is decomposed into its operating regimes which allow us to design different kinds of fuzzy observers. For the sliding mode observer, the reconstruction of vector system states is accomplished by the application of the equivalent output injection concept. An application to sensor fault diagnosis based on the synthesis of the proposed fuzzy observer is realized. The numerical simulation results for biological nutrient removal process show that sensor fault detection can be performed as well by using this type of observer.
References


